Workshop: Analysis of PDEs in Dalian

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Dalian University of Technology, Dalian, China

Title and Abstract -- Talks

Well-posedness Results for Boundary Value Problems Arising in Astrophysical Plasmas

Alonso-Orán, Diego

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Abstract Magnetohydrodynamics (MHD) provides an effective description of electrically conducting fluids and plays a central role in plasma physics, astrophysics, and magnetic confinement fusion. In this talk, I will introduce the magneto-hydrostatic equations, explain their connection with the steady incompressible Euler equations, and discuss several natural boundary value problems arising in this setting. I will present some results on the solvability and well-posedness of these magneto-hydrodynamic boundary value problems, focusing on the key ideas behind the proofs rather than on technical details.

When Viscosity Fails to Select: Three Non-uniqueness Mechanisms for the Navier-Stokes Equations

Cheskidov, Alexey
Westlake Univeristy, China

Abstract Recent work shows that non-uniqueness for fluid equations is not confined to inviscid regimes or rough weak solutions. In this talk, I will discuss three recent non-uniqueness mechanisms that require viscosity and occur for solutions that can enjoy critical or even subcritical regularity. The first produces non-unique dissipative solutions for data that admit a local smooth solution, showing that Lions' weak-strong uniqueness principle for the Euler equations breaks down in the viscous setting. Second, there exist singular stationary solutions in Besov spaces of arbitrarily negative smoothness, showing that non-uniqueness can occur even in subcritical spaces: a perturbative mild solution can arise from an initial datum that is itself a stationary solution. Third, for any smooth (or even zero) initial data, there are infinitely many weak solutions belonging to the Koch-Tataru path space and enjoying the same regularity properties as Koch-Tataru solutions. These non-unique solutions are created by a complete energy cascade from infinite wavenumber, leading to instantaneous blow-up. Again, viscosity is essential. The unifying theme is that viscosity not only regularizes, but also opens new mechanisms for non-uniqueness.

Instantaneous Blowup from Inverse Energy Cascade and Convex Integration

Dai, Mimi

University of Illinois at Chicago, USA

Abstract It is known that one can construct wild solutions for certain type of nonlinear PDEs using the convex integration method. It is curious whether convex integration can be applied to construct blowup solutions. We will discuss recent constructions that give instantaneous blowup for fluid equations using convex integration together with inverse energy cascade mechanism.

On the High Viscosity Limit for the Compressible Navier-Stokes Equations

Danchin, Raphaël

University Paris-Est Créteil, France

Abstract We investigate the high viscosity limit (also called inertial limit) of the barotropic compressible Navier-Stokes equations supplemented with initial data which are perturbations of a stable constant solution.

After suitable rescaling, we establish that the density tends strongly to the solution of a transport-like equation with nonlocal and nonlinear damping term.

In the particular case of constant viscosity coefficients, the limit equation turns out to be globally well-posed, and to have some similarities with the porous medium equation.

Around Yudovich Theory for the Incompressible Euler Equations

Fanelli, Francesco

Basque Center for Applied Mathematics, Spain

Abstract This mini-course revolves around the celebrated Yudovich theory for the two-dimensional incompressible Euler equations in vorticity formulation.

After shortly revisiting the classical result and its proof, we devote attention to the possibility, or impossibility, of extending this theory to other models of inviscid fluids, especially those which are characterised by the presence of some sort of heterogeneity.

In particular, we will discuss two main results:

1. existence and uniqueness of Yudovich-type solutions to the non-homogeneous incompressible Euler equations, under an *a priori* control on a suitable geometric quantity;
2. well-posedness at Yudovich level of regularity of a reduced model for micropolar fluids without viscosity.

Stable and Unstable Interface Dynamics for Gravity Stokes Flow

Gancedo, Francisco

University of Seville, Spain

Abstract We investigate the evolution of the interface between two incompressible fluids of different densities in the Stokes regime under the effect of gravity. We stablish results on global-in-time regularity, stability, and instability.

A Variational Approach to Kinetic Equations

He, Zihui
Bielefeld University, Germany

Abstract In non-equilibrium thermodynamics, GENERIC (the General Equation for Non-Equilibrium Reversible-Irreversible Coupling) describes a broad class of evolution equations for systems exhibiting both reversible and irreversible dynamics. It is formulated in terms of a conserved energy functional, a monotonically increasing entropy functional, and two geometric structures that govern the Hamiltonian and dissipative parts of the dynamics.

In this talk, we present the GENERIC structure of fuzzy Boltzmann equations via a variational characterisation. By fuzzy, we mean that particles interact through delocalised collisions, which can be viewed as an approximation of the inhomogeneous Boltzmann equation. We also study the grazing-collision limit of the fuzzy Boltzmann equation within this variational framework, and show that the limiting Landau equation likewise admits a GENERIC structure.

Stability of Traveling Waves on the Half-plane without Sign Condition on the Vorticity

Jeong, In-Jee
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Abstract We establish Lyapunov stability of a family of traveling waves without the sign condition on the vorticity, for the two-dimensional incompressible Euler equations on the half-plane. This is done by combining variational principles with Lagrangian bootstrapping arguments which carefully track the location of the negative part of the vorticity. This seems to be the first stability result which does not require the sign condition in the half-plane. This is based on joint works with Ken Abe, Kyudong Choi, Guolin Qin, and Yao Yao.

Phase-field Models for Two-phase Flows with Moving Contact Lines, Variable Contact Angles and Bulk-surface Interaction

Knopf, Patrik

University of Regensburg, Germany

Abstract The Cahn-Hilliard equation is the most common model to describe phase separation processes in a mixture of two materials. Moreover, it is further used to describe different phenomena where the distribution and/or motion of two (or more) immiscible materials is considered.

Standard Cahn-Hilliard models are usually endowed with homogeneous Neumann boundary conditions for both the phase-field variable and the chemical potential. However, these boundary conditions yield certain limitations:

1. The diffuse interface separating the materials is enforced to intersect the boundary at a perfect angle of ninety degrees, which is unrealistic in many applications.
2. No transfer of material between bulk and boundary is allowed and thus, absorption process cannot be described.

For these reasons dynamic boundary conditions for the Cahn-Hilliard equation have been introduced. We take a closer look at dynamic boundary conditions that also exhibit a Cahn-Hilliard type structure.

To describe the evolution of two-phase flows, Navier-Stokes-Cahn-Hilliard models have become a popular choice. As the standard models are subject to a no-slip boundary condition for the velocity field as well as homogeneous Neumann conditions for the Cahn--Hilliard subsystem, they exhibit the aforementioned limitations and are also not well-suited for describing general moving contact line phenomena. However, these issues can also be overcome by the introduction of suitable dynamic boundary conditions. Such models will be discussed in this talk.

H^∞-functional Calculus for Stokes Operators

Kunstmann, Peer

Karlsruhe Institute of Technology, Germany

Abstract Given a domain $\Omega \subseteq \mathbb{R}^d$, $d \geq 2$, the Stokes operator is self-adjoint and non-negative in L^2 for various boundary conditions and thus has a functional calculus for bounded Borel functions on $[0,\infty)$. In L^q for $q \neq 2$, one can hope establish a functional calculus for functions that are bounded and analytic on open sectors

$$\Sigma\omega := \{z \in \mathbb{C} \setminus \{0\} : |\arg z| < \omega\},$$

i.e. for functions in $H^\infty(\Sigma\omega)$.

We review several methods to prove boundedness of such an H^∞-calculus for Stokes operators.

In particular, we present a method develpoed in collaboration with L. Weis (KIT, Karlsruhe) that allows to transfer boundedness of the H^∞-calculus for Laplace operators to Stokes operators with corresponding boundary conditions. This method is well-suited for domains with low regularity and we give several examples, including a recent result obtained in collaboration with P. Tolksdorf (KIT, Karlsruhe)

Nonlinear Asymptotic Stability and Optimal Decay Rate Around the Three-Dimensional Oseen Vortex Filament

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Chinese Academy of Sciences, China

Abstract In the high-Reynolds-number regime, this work investigates the long-time dynamics of the three-dimensional incompressible Navier–Stokes equations near the Oseen vortex filament. The flow exhibits a strong interplay between vortex stretching, shearing, and mixing, which generates ever-smaller spatial scales and thereby significantly amplifies viscous effects. By adopting an anisotropic self-similar coordinate system adapted to the filament geometry, we establish the nonlinear asymptotic stability of the Oseen vortex filament. All non-axisymmetric perturbations are shown to decay at the optimal rate $t^{-\kappa|\alpha|^{1/2}}$. At the linear level, this decay mechanism corresponds to a sharp spectral lower bound $\Sigma(\alpha) \sim |\alpha|^{1/2}$ for the nonlocal Oseen operator $L_{\perp} - \alpha \Lambda_{\perp}$, and we identify an explicit spectral point attaining this optimal bound. Combined with the spectral estimates obtained in [25], our analysis fully resolves the conjecture proposed in [12] concerning the asymptotic scaling laws for the spectral and pseudospectral bounds $\Sigma(\alpha)$ and $\Psi(\alpha)$. These results provide a rigorous mathematical explanation for the shear–mixing mechanism in the vicinity of the 3D Oseen vortex filament.

A Dynamical Approach to Stability of Hamiltonian PDEs

Massetti, Jessica
University of Rome Tor Vergata, Italy

Abstract Nonlinear dynamical systems both in finite and infinite dimension are a fundamental instrument to understand/modelize physical phenomena, which often have recurrent/undulatory nature: the rotation of a satellite, the behavior of a planetary system, the motion of the sea, the deflection of a beam, the electromagnetic waves (light, radio waves)...Many of these are modeled by Hamiltonian differential equations (ODEs in finite dimension or PDEs in the infinite case) and their mathematical description is often extremely complicated, characterized by a non-trivial interplay between stable and chaotic behaviors.

A paradigmatic approach consists in studying the existence, robustness and genericity of invariant manifolds that support a global dynamics which can be explicitly described. In the nearly integrable finite dimensional case, these objects are typically tori, have almost full measure, and support a Kronecker flow; this is a rather well established subject. In the infinite dimensional setting, very little is known and the results that one can obtain are strongly related to the boundary conditions of the PDE at stake.

In this short course, I shall discuss the problem of stability of solutions for those PDEs that reveal an evolutionary equation structure. These ones benefit of a dynamical system approach that aims at pulling the system of (infinitely many) equations back to a suitable normal form, from which one can deduce the dynamical behavior of solutions, either in terms of perpetual stability for a special bunch of initial data evolving on invariant tori, or of long time stability for open sets of initial conditions.

A common denominator in these two cases, relies on the solvability of homological equations and the so-called problem of small divisors.

Computer-assisted Existence and Multiplicity Proofs for Semi-linear Elliptic Boundary Value Problems

Plum, Michael

Karlsruhe Institute of Technology, Germany

Abstract Many boundary value problems for semi-linear elliptic partial differential equations allow very stable numerical computation of approximate solutions, but are still lacking analytical existence proofs. In the lecture, a method is proposed which exploits the knowledge of a "good" numerical approximate solution in order to provide a rigorous proof of existence of an exact solution close to the approximate one. This goal is achieved by a Newton-Kantorovich-type fixed-point argument which takes all numerical errors into account, and thus gives a mathematical proof which is not "worse" than any purely analytical one. A significant role in this fixed-point argument is played by eigenvalue bounds for the linearization of the given boundary value problem at the approximate solution. The method is used to prove existence and multiplicity statements for many specific examples of boundary value problems on bounded and on unbounded domains, with focus on cases where purely analytical methods had not been successful.

On the Lions Density Patch Problem for the Inhomogeneous Incompressible Navier-Stokes Equations

Tan, Jin

Chinese University of Hong Kong, China

Abstract We consider the incompressible Navier–Stokes equations with variable density. Global-in-time existence of finite-energy weak solutions was established by P.-L. Lions in 1996 without assuming a positive lower bound on the initial density, thereby allowing for vacuum. Since then, the persistence of boundary regularity of a density patch has remained an open problem. In the case of a bounded fluid domain and constant viscosity, Lions' density patch problem is now well understood. In this talk, I will discuss the whole-space case and present space-time weighted estimates for the velocity field, which provide a partial answer to Lions' problem. In the end, some new ideas for the case of density-dependent viscosity will be discussed. This talk is based on joint works with Christophe Prange (Cergy Paris University).

The Motion of a Rigid Body in an Inviscid Compressible Fluid

Wiedemann, Emil

Friedrich-Alexander University Erlangen-Nürnberg, Germany

Abstract We study a fluid-structure interaction problem where a rigid body is immersed in an inviscid compressible fluid. The dynamics is thus modelled through a coupling between the rigid motion and the compressible Euler equations, which in itself pose considerable mathematical challenges. We show that a generalised form of solution (a so-called dissipative measure-valued solution) can be obtained as a vanishing viscosity limit from the compressible Navier-Stokes system with Navier boundary conditions, and that the solution enjoys the property of weak-strong uniqueness. To our knowledge, this is the first rigorous result on inviscid compressible fluid-structure interaction. Joint work with Qianfeng Li (Erlangen).

The Quartic Integrability of Water Waves and Related Models

Wu, Sijue
University of Michigan, USA

Abstract TBA

Infinite-in-time Growth in 2D and 3D Euler Equations

Yao, Yao

National University of Singapore, Singapore

Abstract In this talk, I will discuss two recent results on infinite-in-time growth in 2D and 3D incompressible Euler equations. For the 2D Euler equation on the whole plane, we construct the first example giving superlinear growth of the vorticity gradient for smooth compactly supported vorticity (joint with In-Jee Jeong and Tao Zhou). For the 3D axisymmetric Euler equation without swirl, we establish some upper and lower bound for the radial moment of vorticity, and prove that under some sign and symmetry conditions, all solutions must have their vorticity L^q norm growing to infinity with some power-law rate for all $p \geq 1$ (joint with Khakim Egamberganov).

Global Nonlinear Stability of Vortex Sheets for the Navier-Stokes Equations with Large Data

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Abstract This paper concerns the global nonlinear stability of vortex sheets for the Navier-Stokes equations. When the Mach number is small, we allow both the amplitude and vorticity of the vortex sheets to be large. We introduce an auxiliary flow and reformulate the problem as a vortex sheet with small vorticity but subjected to a large perturbation. Based on the decomposition of frequency, the largeness of the perturbation is encoded in the zero modes of the tangential velocity. We discover an essential cancellation property that there are no nonlinear interactions among these large zero modes in the zero-mode perturbed system. This cancellation is owing to the shear structure inherent in the vortex sheets. Furthermore, with the aid of the anti-derivative technique, we establish a faster decay rate for the large zero modes. These observations enable us to derive the global estimates for strong solutions that are uniform with respect to the Mach number. As a byproduct, we can justify the incompressible limit.